properties (without adjustable constants) when scattering is acutely anisotropic, it is necessary to resort to either exact numerical methods or one of the more complicated approximate models (three-flux, six-flux, etc.).

REFERENCES

1. S. Chandrasekhar, *Radiative Transfer*. Dover, New York (1960).
2. A. Schuster, Radiation through a foggy atmosphere, *Astrophys. J.* **217**, 1–22 (1905).
3. K. Schwarzchild, Equilibrium of the Sun's atmosphere, *Ges. Wiss. Gottingen, Nachr., Math.-Phys. Klasse* **1**, 41–53 (1906).
4. H. C. Hamaker, Radiation and heat conduction in light scattering material, *Philips Res. Rep.* **2**, 55–67, 103–111, 112–125, 420 (1947).
5. K. Klier, Absorption and scattering in plane parallel turbid media, *J. Opt. Soc. Am.* **62**, 882–885 (1948).
6. P. Kubelka, New contribution to the optics of intensity light-scattering materials, Part I, *J. Opt. Soc. Am.* **38**, 448–457 (1948).
7. B. F. Armaly and T. T. Lam, Influence of refractive index on reflectance from a semi-infinite absorbing-scattering medium with collimated incident radiation, *Int. J. Heat Mass Transfer* **18**, 893–899 (1975).
8. C. Sagan and J. Pollack, Anisotropic nonconservative scattering and the clouds of Venus, *J. Geophys. Res.* **72**, 469–474 (1967).
9. G. Domoto and W. Wang, Radiative transfer in homogeneous non-gray with nonisotropic particle scattering, *J. Heat Transfer* **96**, 385–390 (1974).
10. B. K. Larkin and S. W. Churchill, Heat transfer by radiation through porous insulations, *A.I.Ch.E. Jl* **5**, 467–473 (1959).
11. J. C. Chen and S. W. Churchill, Radiant heat transfer in packed beds, *A.I.Ch.E. Jl* **9**, 35–41 (1963).
12. C. F. Sanders and J. M. Lenoir, Radiative transfer through a cloud of absorbing-scattering particles, *A.I.Ch.E. Jl* **18**, 155–160 (1972).
13. K. J. Daniel, N. M. Laurendeau and F. P. Incropera, Prediction of radiation absorption and scattering in turbid water bodies, *J. Heat Transfer* **101**, 63–67 (1979).
14. T. W. Tong and C. L. Tien, Analytical models for thermal radiation in fibrous insulations, *J. Thermal Insulation* **4**, 27–44 (1980).
15. H. C. Hottel and A. F. Sarofim, *Radiative Transfer*. McGraw-Hill, New York (1967).
16. A. Dayan and C. L. Tien, Radiative transfer with anisotropic scattering in an isothermal slab, *Jl Quantve Spectros. & Radiat. Transf.* **16**, 113–125 (1976).
17. T. W. Tong and C. L. Tien, Resistance network representation of radiative heat transfer with particulate scattering, *Jl Quantve Spectros. & Radiat. Transf.* **24**, 491–503 (1980).
18. Q. Brewster, Radiative transfer in packed and fluidized beds, Ph.D. Dissertation in Mechanical Engineering, University of California, Berkeley (1981).
19. M. Q. Brewster and C. L. Tien, Radiative transfer in packed/fluidized beds: dependent vs. independent scattering, *J. Heat Transfer*, to be published (1982).
20. H. C. Hottel, A. F. Sarofim, L. B. Evans and I. A. Vasalos, Radiative transfer in anisotropically scattering media: allowance for fresnel reflection at the boundaries, *J. Heat Transfer* **90**, 56–62 (1968).
21. R. Viskanta, A. Ungan and M. P. Menguc, Predictions of radiative properties of pulverized coal and fly-ash polydispersions, ASME Paper 81-HT-24 (1981).

Int. J. Heat Mass Transfer. Vol. 25, No. 12, pp. 1907–1909, 1982
Printed in Great Britain
0017-9310/82/121907-03 $03.00/0

A STEADY STATE LINEAR ABLATION PROBLEM

G. J. WEIR

Applied Mathematics Division, Department of Scientific and Industrial Research, Wellington, New Zealand

(*Received* 27 *January* 1982)

NOMENCLATURE

h,	heat transfer coefficient;
r,	rod radius;
r',	radial coordinate;
r_s,	maximum ablation radius, equation (12);
s,	rod resistivity;
z,	axial coordinate;
C,	rod heat capacity;
I,	current;
I_s,	maximum rod current, equation (11);
J,	energy flux;
K,	rod thermal conductivity;
T,	temperature;
$T(0)$,	facial temperature;
T_0,	external temperature;
T_s,	sublimation temperature;
T_∞,	asymptotic rod temperature;
U,	rod speed;
U_s,	maximum rod velocity, equation (10);
V,	potential drop;
V_∞,	potential for constant rod temperature.

Greek symbols

λ,	latent heat of vaporisation;
μ,	non-ablating inverse distance;
ν,	ablating inverse distance;
ρ,	rod density;
ϕ,	angular coordinate.

INTRODUCTION

CONSIDER a semi-infinite cylindrical rod of radius r lying along the positive z axis of a cylindrical coordinate system (r', ϕ, z). If a uniform distribution of current of I A enters the rod through the surface $z = 0, 0 \le r' \le r$ (henceforth called the face), evaporating material from the face, with what speed U must the rod move toward the face so that the ablating face remains at $z = 0$?

We shall assume that radial temperature variations may be ignored, and that heat losses from the rod to an environment (occupying $z < 0; r' > r, z > 0$) at temperature T_0 satisfy a linear heat transfer law. This problem idealises a technique considered for acetylene production by an electric arc [1].

If $T = T(z)$ is the (steady) temperature of the rod, then the energy flux J along the rod satisfies

$$J = -K\frac{dT}{dz} - \rho CUT$$

where K is thermal conductivity, ρ density, C heat capacity, and so

$$\frac{dJ}{dz} = -K\frac{d^2T}{dz^2} - \rho CU\frac{dT}{dz} = \frac{sI^2}{\pi^2 r^4} - 2h(T - T_0)/r \quad (1)$$

where the source term results from Joule heating, the sink term from heat transfer to the environment, s denotes resistivity, and h the heat transfer coefficient. We have assumed that r, U, T_0, K, ρ, C, s and h are constants.

As boundary conditions, temperature is a constant T_s at $z = 0$, where T_s is the sublimation temperature of the rod,

$$T = T_s \text{ at } z = 0; \quad (2)$$

while for large z, longitudinal temperature gradients approach zero, or from equation (1),

$$T = T_0 + sI^2/(2\pi^2 r^3 h) \quad (3)$$
$$= T_\infty \quad \text{as } z \to \infty.$$

Finally, U follows from an energy balance at the face. Assuming that the current entering the face undergoes a constant potential drop V (perhaps equal to the sum of the anode drop and the work function), then at the face the input electrical work per unit area is $IV/(\pi r^2)$, which must equal the corresponding heat transfer loss $h(T_s - T_0)$ plus the vaporisation loss $\lambda\rho U$ [where λ is the (constant) latent heat of vaporisation] plus the conduction loss $-K(\partial T/\partial z)$ minus the input mass flux factor of $\rho C T_s U$, or

$$IV/(\pi r^2) = h(T_s - T_0) + \lambda\rho U - K\frac{dT}{dz} - \rho C T_s U \text{ at } z = 0. \quad (4)$$

Equations (1)–(4) define our steady state linear ablation problem.

THE NON-ABLATING PROFILE

For ablation to begin the face must be at the sublimation temperature T_s, which from equation (4) is impossible for sufficiently small currents. When the facial temperature is below the sublimation temperature then equation (2) is ignored, and equation (4) replaced by

$$IV/(\pi r^2) = h(T - T_0) - K\frac{\partial T}{\partial z} \text{ at } z = 0. \quad (5)$$

The solutions of equations (1), (3) and (5) are

$$T = T_\infty + I(V - V_\infty)e^{-\mu z}/[\pi r^2(h + K\mu)],$$
$$\mu = (2h/Kr)^{1/2},$$
$$V_\infty = sI/(2\pi r),$$

where V_∞ is the potential drop (with I fixed) which produces a constant cylinder temperature of T_∞. The facial temperature $T(0)$ satisfies

$$T(0) = T_0 + \frac{IV + sI^2K\mu/(2\pi rh)}{\pi r^2(h + K\mu)}$$

and so ablation begins when $T(0) = T_s$, or

$$I \ge \frac{\pi rhV}{sK\mu}\left\{\left[1 + \frac{2(T_s - T_0)(h + K\mu)sK\mu r}{hV^2}\right]^{1/2} - 1\right\}. \quad (6)$$

THE ABLATING PROFILE

For currents satisfying equation (6), equations (1)–(3) are valid, and the temperature profile is

$$T = T_\infty + (T_s - T_\infty)e^{-vz}, \quad (7)$$
$$v = \{[(\rho CU)^2 + 8hK/r]^{1/2} + \rho CU\}/2K. \quad (8)$$

In practice $T_\infty < T_s$, since otherwise the cylinder will explode [2] due to the radial temperature gradient initiating ablation *inside* the rod along its axis. We shall assume that $T_\infty < T_s$ in what follows, and continue to ignore radial temperature gradients since these cause difficulties in defining the boundary condition on the ring $z = 0$, $r' = r$. We also assume $\lambda > CT_s$, since otherwise the cylinder could not satisfy equation (2).

Perhaps the most interesting result is that the steady speed U can be found exactly [from equations (4), (7) and (8)] from the quadratic equation

$$\rho^2(\lambda - CT_s)(\lambda - CT_\infty)U^2 - 2\rho[IV/(\pi r^2) - h(T_s - T_0)]$$
$$[\lambda - (C/2)\times(T_s + T_\infty)]U + [IV/(\pi r^2) - h(T_s - T_0)]^2$$
$$- (2hK/r)(T_s - T_\infty)^2 = 0, \quad (9)$$

which solves for a type of 'adjustable moving boundary' problem.

For conciseness, we shall state some easily derived results in this paragraph. The 'negative sign' is required in the standard solution of the quadratic equation in equation (9). The condition $U = 0$ in equation (9) is equivalent to equality in inequality equation (6). The discriminant of equation (9) is zero if $T_\infty = T_s$, and so equation (9) guarantees positive U for physically reasonable parameter values. Finally, during ablation U is a monotonically increasing function of I, and as T_∞ increases monotonically with I, the maximum physical speed U occurs when $T_\infty = T_s$.

When $T_\infty = T_s$, the corresponding speed U_s is

$$U_s = \frac{I_sV/(\pi r^2) - h(T_s - T_0)}{\rho(\lambda - CT_s)} \quad (10)$$

where

$$I_s^2 = 2\pi^2 r^3 h(T_s - T_0)/s. \quad (11)$$

A quantity of physical interest is the (maximum) rate of ablated mass loss, $\pi r^2\rho U_s$. From equation (10), r^2U_s increases as $r^{3/2}$ [since I_s does from equation (11)] but decreases as r^2, and so by allowing only r to vary, r^2U_s increases with r from zero to some maximum, and then decreases to zero since the increasing heat losses from the face will eventually force the facial temperature below T_s and ablation will cease. The maximum value of r^2U_s occurs when

$$r = r_s = 9V^2/[8sh(T_s - T_0)], \quad (12)$$

and ablation ceases when

$$r > 16r_s/9. \quad (13)$$

Of course it is necessary to check that radial temperature variations are unimportant for equations (12) and (13) to hold, and a standard calculation involving asymptotic radial temperature variations suggests the condition $sl^2/(4\pi^2 Kr^2 T_x) \ll 1$.

Finally, since many of the parameters determining ablation problems are poorly determined, the theory above may supply useful estimates for certain engineering requirements.

Acknowledgements—The author is grateful to Dr Clive Davies and Dr John Abrahamson for communications on various aspects of ablation.

REFERENCES

1. J. Abrahamson, C. Davies, J. Stott, R. Ward and P. Wiles, Erosion rates of graphite anodes in high current arcs, *I. E. C. Fundamentals*, **19**, 233–243 (1980).
2. C. Davies and J. Abrahamson, 'Limit to erosion rate in a high current carbon arc', *I.E.C. Process Des. Dev.* to be published.

Int. J. Heat Mass Transfer. Vol. 25, No. 12, pp. 1909–1911, 1982
Printed in Great Britain
0017-9310/82/121909-03 $03.00/0

ANALYSIS OF NUSSELT-TYPE CONDENSATION ON A TRIANGULAR FLUTED SURFACE

C. B. PANCHAL
Argonne National Laboratory, Argonne, IL 60439, U.S.A.

and

K. J. BELL
School of Chemical Engineering, Oklahoma State University, Stillwater, OK 74074, U.S.A.

(*Received* 19 *October* 1981 *and in final form* 9 *April* 1982)

NOMENCLATURE

- a, amplitude of flute [m];
- A, ratio of flute amplitude to pitch, a/p;
- g, gravitational constant [m s^{-2}];
- h_w, conductance of wall [W m^{-2} K^{-1}];
- H_v, latent heat of vaporization [J kg^{-1}];
- K, thermal conductivity [W m^{-1} K^{-1}];
- Nu, Nusselt number, Up/K_f;
- p, period (pitch) of flute [m];
- Re, Reynolds number, $4w/(X_L\mu)$;
- S, thickness of unfluted zone of tube wall [m];
- T, temperature [K];
- U, average overall heat-transfer coefficient [W m^{-2} K^{-1}];
- w, axial mass flow rate of liquid [kg s^{-1}];
- X_L, perimeter of half-flute [m].

Greek symbols

- δ, condensate-film thickness [m];
- δ_0, condensate-film thickness at midpoint of meniscus [m];
- λ, dimensionless physical property group, $(4\rho^2/\mu^2)g\delta_0^4/X_L$;
- μ, viscosity of condensate [Pa s];
- ρ, density of condensate [kg m^{-3}];
- σ, surface-tension coefficient [N m^{-1}];
- Ω, dimensionless physical property group, $K_f(T_v - T_c)/(\sigma H_v ap)$.

INTRODUCTION

CONDENSATION on a fluted surface and the resulting enhancement in the heat transfer coefficient was first recognized by Gregorig [1]; however, the concept was not fully applied until recently. Various theoretical studies [2–7] and experimental studies [8–12] show that the phenomena governed by surface tension can enhance the condensate-film coefficient. In a previous study in which condensation on a vertical 'cosine' fluted surface was analyzed, wall resistance was assumed to be negligible [2]. Edwards *et al.*, in their analysis of condensation on a horizontal tube with transverse flutes, mentioned the importance of the wall resistance [3]. Fuji and Honda solved, for a cosine flute, the difficult set of equations that numerically describes heat transfer in the condensate film and the tube wall, and they introduced the concept of a representative value for wall thickness [4]. However, the importance of wall resistance is not yet fully understood. The present study analyzes condensation on a vertical 'triangular' fluted surface and also investigates the importance of the wall resistance.

DESCRIPTION OF THE PROBLEM

Figure 1 shows condensation on a vertical fluted surface. The figure also defines three coordinate systems (a cylindrical system with a vertical z axis and two rectangular systems) as well as some important geometric parameters.

Unlike condensation on a smooth vertical surface, condensate film on a fluted surface in two directions: vertically (due to gravity) and horizontally from the crest to the low point of the trough (due to the surface-tension force). As a result, the condensate accumulates in the trough, leaving only a very thin film near the crest. The heat transfer coefficient of the condensate film is large near the crest, while it is small at the low point of the trough. This results in a circumferentially nonuniform heat transfer flux in the tube wall. It is expected that the temperature at the condensate–wall interface will vary from almost saturation temperature, T_v at the crest to some lower value at the bottom of the trough. It is also expected that heat fluxes at the coolant–wall interface will be, in general, circumferentially nonuniform, although the coolant heat transfer coefficient is assumed to be constant.

An exact analysis of this problem could be difficult, if not impossible. However, previous work [2] leads to some assumptions that simplify the problem somewhat. The principal assumption used in this analysis is that horizontal cross flow is negligible at the low point of the trough where the film is thick, while the vertical flow is negligible near the crest.